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Book Review

Of Biocultural Mathematics and Mind

Reflections On and Around *The Origin and Evolution of Cultures* by Robert Boyd and Peter J. Richerson. New York: Oxford University Press, 2005.

Charles J. Lumsden, Room 7313, Medical Science Building, University of Toronto, 1 King's College Circle, Toronto, Ontario, Canada M5S 1A8. Email: charles.lumsden@utoronto.ca.

My comments on and around this fine book proceed from three basic premises. If you do not share them, be advised there is little to follow that you will find reassuring or informative, except perhaps as filler for a void in chat when you're next together with associates who, like yourself, also have little use for evolutionary science, mathematics, and their connections. So please be warned. Time is precious; I have no interest in wasting yours.

My first premise is that evolution, including the biological, genetic evolution of our human species, is a fact. Thus, what follows is not a defense of evolutionary thinking and why it is important in coming to terms with human beings and the societies they form. Nor is it an argument against the alleged attacks on evolutionary science by people with beliefs attributing human origins to divine intervention or to the actions on our universe of beings identical to (or, for us, indistinguishable from) gods. If you are of the former school and find surprising the assertion that humans, like all life on Earth, are evolved organic beings, then as far as I am concerned you have either been asleep the past century or cast away on a very remote island. If you adhere to the latter belief system and see the proper understanding of humankind as a matter for priests rather than scientists, then you have my respect, and my best wishes for a life good to yourself and others. But you will find little of value in what follows.

The fact of human evolution (taken here as self-evident) is of course not the same as the understanding of how that evolution occurred (still a scientific mystery, largely unexplained), It is also not the same as knowing why this evolution occurred the way it did over the millions of years needed to transform our hominid ancestors into us, rather than taking place some other possible way. Scientific conjectures about how human evolution occurs, and why our branch of the hominid evolution tree flourished while those nearby withered, have always attracted plenty of attention. Human beings find themselves interesting. Since the ratio of conjectures to hard data has traditionally been quite high, the debate surrounding these conjectures has gone on for a long time.

Polarizing this range of conjectures has been the importance of evolved biological (read "genetic") elements in understanding human nature. One extreme position sees genetic change nudging the bodies, brains, and minds of ancestral hominids over an evolutionary threshold, into human form, and then "switching off" as cultural innovation and social evolution took over. Another extreme sees in the data a densely innate pattern of biologically evolved drives and needs wired into the brain right up to the present day, a stone age genetic strait jacket from which we cannot escape. If you are not a fan of simple alternatives, you can pick a comfortable spot somewhere between these two extremes and wait for more data to roll in.

Thirty years ago last year, the eminent Harvard evolutionary biologist Edward O. Wilson took a profoundly more subtle and complex approach to the question of behavior and psyche in human beings and, indeed, in all social animals. In a pair of landmark books – *Sociobiology: The New Synthesis* (1975) and *On Human Nature* (1978) – Wilson explored the diverse literatures bearing on human history, psychology, and social life, concluding that Darwinian genetic evolution cannot be ignored if we are to grasp what made us human. The whirl of controversy which quickly enveloped the human sciences in answer to Wilson's books is itself the focus of a growing literature (Segerstråle, 2000). For our purposes here, suffice it to say that by the late 1970s, genes, Darwin, and Darwinian evolution were part of a new agenda for understanding the history and psychology of human beings – an agenda to which the subsequent rise of the Human Genome Project and the field of evolutionary developmental genetics ("evo-devo": Carroll, Grenier, and Weatherbee, 2005; Davidson, 2006) has added impressive empirical depth.

The new agenda also marked a time of striking innovation in the way evolutionary hypotheses about human nature were expressed and investigated. This brings us to my second premise, which is that in understanding the evolution of human nature, mathematics matters. That is, we must explore the exciting possibility that explanations of human mental organization, social dynamics, and evolution are irreducibly mathematical in form: that the truth about us is encoded in a mathematics, albeit one with properties still largely undiscovered. Please note that my second premise is not that mathematics *is* essential. The premise is that, given the enormous effectiveness of mathematical reasoning in other sciences, we should *find out* whether or not it is the best way to talk about the human mind, culture, and history.

A premise assigning high priority to mathematics is not, I think, horridly provocative or controversial in itself, at least not any more. Mathematics is everywhere. Perhaps we should thank our currency economies, with all their a-counting. Any disinterested student who has yawned through an introductory economics course has seen the curves of supply and demand intersect at their optimal attractor point. In the areas of serious research, mathematical models and equations have been studied in psychology, sociology, and anthropology throughout the twentieth century (Ball, 2004; Epstein and Axtell, 1996; Fararo, 1978; Hamlin, Jacobsen, and Miller, 1973; and Rashevsky, 1951 provide a small sampling). Even literary theorists are using nonlinear attractors and chaotic dynamics (e.g. Hayles, 1990).

Within evolutionary science, population biology and genetics have received extensive mathematical development over the past century, first in the hands of pioneers like J. B. S. Haldane, Ronald Fisher, and Sewall Wright, then with the axiomatics of the neo-Darwinian synthesis, and most recently with the mathematics of biocultural evolution.. Certainly, the priority of mathematics in getting to the bottom of things in physics, chemistry, and engineering, has been evident for centuries. Try building a television set or airliner without it.

So by the late 1970s, mathematics was “in the air” across the natural and social sciences. You’ll note a missing link, however: a system of mathematical reasoning that would connect the human sciences not just within themselves, but also cross-bridge them to the mathematical structures of the biological/evolutionary sciences and, beyond that, to the rest of the physical and natural sciences as well. Stimulated in part by Wilson’s sociobiology controversy, the search was on for such connections and their ability to predict testable outcomes about human psychological and social evolution.

From time to time new science seems to be the province of the great solitaries, thinkers who, like the White Whale, somehow slip beneath the surface of existence and penetrate deep into reality, far below the realm accessible to the rest of us: Newton, Darwin, Einstein, Hamilton, Goodall. At other times, it seems to belong to huge teams of investigators, such as those whose dedication drives the mapping of whole genomes at the molecular biology institutes or of the world of elementary particles at the giant accelerator labs. Sometimes, though, good things seem to come in (or through) pairs: Louis and Mary Leakey for example, or the brothers Wright, or Watson and Crick, or Cousteau and Gagnan.

For reasons still to be unraveled by historians of science, the mathematical explorations of human biological evolution, cultural evolution, and social history born of those controversies and opportunities of the late 1970s took shape in the work of several pairs of scholars: Ed Wilson and myself (1981) writing from a sociobiological perspective, Luigi Cavalli-Sforza and Marc Feldman (1981) from the standpoint of social networks, Robert Boyd and Peter Richerson (1985) from the direction of population biology and cultural transmission, and Leda Cosmides and John Tooby (1989) from the vantage of axiomatic evolutionary psychology.

The books by Ed Wilson and me, and by Cavalli-Sforza and Feldman, appeared first in the early progression, in 1981. Wilson and I, introducing the term gene-culture coevolution perhaps for the first time, took a mathematical approach to human genes, minds, and culture anchored in developmental psychology. Cavalli-Sforza and Feldman, for their part, mapped the remarkable effects exerted by the directions of information transmission through social networks treated as mathematical patterns. With their work the adjectives *horizontal*, *vertical*, and *oblique* took on new meaning and significance for evolutionary biologists. Boyd and Richerson’s (BR hereafter) influential monograph followed several years later, in

1985, and they have continued with studies that steadily deepened our understanding of the strengths, and limitations of simple mathematical models as probes of human evolution.

In the present book, Boyd and Richerson provide a much needed compilation of key papers marking this further development of their approach. The assembled publications span a period of almost fifteen years, from 1989 through 2003. Each is given its own chapter, for a total of twenty chapters. The set is headed by a concise but wide-ranging introduction that summarizes the history of their collaboration and their progress in this exciting field, along with the core propositions around which their mathematical models are organized.

Scholars new to mathematical modeling are right to wonder if the method's abstract beauties also have meaningful content. Nature seems exuberantly complex; the mathematical models are deliberately simple. Where's the match-up? The standard apologies offered in the face of such justifiable skepticism are relevant to appreciating the importance of books like this one, as well as the literature of which it is a part. So it makes sense to touch on them briefly here. Indeed, a flip through the book under review reveals more than a few formulas and equations, some rather thickly laid onto the pages.

The casual browser who normally spends her time dissecting Chaucer or tracing Fellini's source material might ask if a close reading of this book is worth the time, given all the maths and graphs and charts. It is worth it. The text is well accessible despite the technical nature of its mathematical approach. In the main body of each chapter, equations are used when essential and their terminology is presented in a logical manner and tied directly to the biocultural problem. Although the cadence from time to time favors the applied mathematician, for the most part detailed math is kept in technical appendices.

Since the works assembled, as we shall see in a moment, cover a wide range of topics in biocultural science, the reader is at liberty to sample the chapters or chapter sections as personal interest dictates. There is no need to slog through from first chapter to last, initial equation to final. The sequence chosen for the chapters, however, very nicely showcases the progression of BR's ideas and modeling strategy over the years. The structure of the volume therefore will benefit both the general reader as well as the specialist or the student wishing a strong technical introduction to their methods.

The closer reading will, however, give certain pause, because the equations BR deploy seem strikingly simplistic, at least to a postmodern eye keyed to deeply subtle phenomena of mind and culture. In this book, culture's dense forest of symbols (Turner, 1967) has faded into a thin shimmer of "replicators" housed in formulas bearing abstractions like "frequencies," "adaptive character," and so on. To dismiss such fare as pale beer would, however, be to overlook the astonishing impact simple mathematical readings of Nature have effected, right across the sciences. Philosophers may from time to time moan and groan, but simple works. Sometimes.

The first apology as to why, when it comes to human nature and history, simple mathematical models are worthwhile makes slight extension of the appeal to mathematical beauty. It notes that by taking a mathematical approach, we are forced to clarify our thinking to the point at which specific mathematical terms can be defined and arranged into formulas that connect logically one to another, and to the reasoning apparatus of mathematics as a whole. That may not sound like much, but in practice such formalization (as it is called) helps throw fuzzy thinking about mind and culture into harsh relief, along with the lexical bafflegab and ideative twaddle such thinking comes packaged in. This is not to say suspect notions (memes? culturgens?) cannot be hidden under the mathematical bush – indeed they can be – but the axiomatic structure of math spreads the branches of the bush more thinly than otherwise, making the bafflegab tougher to keep out of sight.

The second apology is an appeal to that scientific favorite, "simpler first." Experimental science uses more or less the same apology, pointing out the virtues of tightly controlled arrangements in which the normal flux of reality is confined to just a few independent variables. Correlations can then be monitored, null hypotheses maybe rejected. In mathematical modeling, it also makes sense to begin with axiomatic bindings among a few variables suspected to be crucial in the relationship of cause to effect. Thus, compared to the real developing human mind or evolving culture, the mathematical model is a stripped down representation. But so, by deliberate intent, is what happens in the experimental laboratory – again, not an unfamiliar scenario.

The goal, of course, is to strip away only the inessential, keeping the gist in a mathematical model with helpful properties, and perhaps even leading to deep insights. The evolutionary theorists of the past century – Haldane, Fisher, Wright, Crow, Kimura, Maynard Smith, Hamilton – racked up impressive gains following this path. It is a cognitive strategy that is not confined to mathematical or experimental science, but suggestive of the informed workings of our everyday attention. In learning to drive a car, for example, we are well advised to concentrate on our control inputs and the rules of the road, not on the color of our roadster's paint job or the metallurgy of its engine block. In this apology the mathematical model is not the whole truth, nor is it nothing but the truth. The gist expressed by its few formal terms is intended as a telling caricature.

Many mathematical models, including those studied in the book, are offered in this spirit. Indeed, in their Chapter 19, entitled "Simple models of complex phenomena: The case of cultural evolution" BR wrestle with the problem from a stance quite similar to the one I've outlined in the preceding paragraph. They introduce some specialized terminology ("generalized sample theories," "modularization of analysis") to help make the point, but if I read them at all accurately it boils down to "go for the gist."

A third apology for why apparently simple math models can work very well is more abstruse and less familiar, but also of potentially deep significance. It is the appeal to so-called *universality* and so far has been worked mostly within the physical sciences and by physical scientists who are interested in complex biological systems.

Universality is a bold epistemic position on the organization of nature in relation to mathematics, including human nature and biosocial evolution. Essentially, a universality hypothesis says the world is organized into categories or classes in which apparently simple and apparently complex phenomena belong together. They do so because, deep down, they all follow identically the same simple mathematical rules, at least in certain key conditions, despite their differences in apparent complexity.

Notice how different this apology for math is, compared to the natural appeal of "simplest first" as outlined above: under universality, the parsimonious mathematical formulation is not a first, crude caricature of a more subtle, intricate reality. It is the *exact* formal description of that reality, shared identically by the simple mathematical model and the actual, real-world system. The discovery, for example, that real gene-culture coevolution (GCC for now) works in exactly the same way as the hugely simple GCC mathematical models studied to date would be an example of universality in action. I'm not aware of any such universality proof or measurement for human GCC models as yet, but there is already some hint of the idea's potential relevance. For example, Geoff Clarke and I (2005a,b) have recently found signs of universality in the developmental mathematics of cell death in neural cell populations in the brain and peripheral nervous system, across a range of species and diverse developmental conditions.

A fourth apology for mathematics, the appeal to vulgar reductionism, merits a quick look. In using the term "vulgar" I show my age as a child of the 1960s and 70s, perhaps, but my purpose in doing so is as follows. A science of GCC must connect events of diverse kinds: signaling among genes drives neurogenesis which sculpts nerve cell circuits that are shaped by experience that responds to cultural setting that influences survival, reproduction, artifact production, and so on. Mathematical treatments of GCC are therefore "consilient" (Wilson, 1998) or "holistic" (Lumsden, 1997) insofar as such diverse elements draw together into meaningful patterns.

These connections are not "vulgar reductionism" (VR). VR is the claim that notions like cognition or the collapse of a civilization are of no scientific merit, and should be cast out. To VR they are epi-phenomena, nothing but fuzzy minded stand-ins for the molecules comprising the creatures and ecosystems. VR wants explanations via the molecules and atoms. A VR modeler might insist that mathematics is the optimal language of GCC because, to explain biocultural evolution scientifically, we just need get all the right DNA molecule data etc. into the biggest computer and solve the molecular equations. Then absolutely everything can be predicted, completely and rigorously, through those solutions.

A point of view like VR seems far fetched because we really do not have computers this powerful, or databases of information so complete that we can write down all of the molecules in a nerve cell or a collapsing civilization. Given the pace of advance in computer engineering and the empirical mapping of cells and tissues in molecular terms, however, it might not be outlandlish to contemplate a day when we do have this much information and so undertake simulations of this kind. I think, therefore, that appeals to current empirical ignorance will not quite do in seeing what is really wrong with VR as a context for the mathematics (or anything else). For, even if successful, a VR simulation succeeds only in tracking the molecules. Watching only molecules, it has nothing to say about us in terms of cells, brains, humans, or cultures.

I take it as uncontroversial, however, to say in present company that science is about understanding as well as prediction. If so, then we have no reason to suspect that making sense of human nature and its evolution is possible (for brains and minds like ours) with a science-speak in which the vocabulary is just molecules or elementary particles. Cells, brains, humans, and cultures are pivotal way points in our consilient map of human science. We don't want to get rid of them in favor of molecules or quarks. The only thing a VR model and simulation can show us in GCC is that nonrelativistic quantum mechanics is an accurate theory of molecules under the conditions prevailing on the surface of our planet. This we already know, however, to enormous accuracy, from physics itself. The authors of this important book rightly ignore VR, as do all practitioners of GCC mathematics of whose work I'm aware. The one possible exception is Roger Penrose's hotly debated conjecture that human consciousness is a non-computable natural phenomenon (Penrose, 1994). I shall return to this briefly below.

The book's twenty chapters are organized into five groups of mathematical applications carried out using simple models: The Evolution of Social Learning (five chapters), Ethnic Groups and Markers (two chapters), Human Cooperation, Reciprocity, and Group Selection (seven chapters), Archaeology and Culture History (three chapters), and Links to Other Disciplines (three chapters). To a reader like myself, who spends a lot of time wondering about mind and culture, all these sections are strikingly useful because of the clarity of their exposition and the potential significance of their results. More general readers, interested in what culture change and biocultural evolution are, and what theories of these phenomena look like, also will find the entire book highly relevant. BR's explanations of the relative likelihood of cultural evolution *per se*, of social learning as an evolved strategy, and of the novel pathways to cooperation and multi-level evolution open to culture-bearing creatures, will deservedly attract further attention now that they are available via this well organized compilation.

BR have done us all a favor by using their Introduction to state, and discuss, the propositions at the heart of their work. They are five. Some, such as "*Genes and culture coevolve*", that "*Culture is part of human biology*", and that "*Culture makes human evolution very different from the evolution of other organisms*", will sit happily with a lot of people, including myself. The other propositions less so. Their second one, that culture should be modeled as a Darwinian evolutionary process, strikes me as just a retread of Don Campbell's old "selection/retention" culture change model and an excuse to hide a lot of important questions about the mind inside empty parameters about preference and utility. This may have been best practice at one time, but I think we now need to do better, for reasons I'll get to in a moment.

Their first proposition, "*Culture is information that people acquire from others by teaching, imitation, and other forms of social learning*" really will not do any more either; it just begs the question of what we mean by information, social, learning, acquire and so on in such work. The 1980s attitude of "*I knows it when I sees it*" does not meet our need for evolutionary-mathematical approaches that say more about the interior life of culture and of mind. A little earlier I alluded to Victor Turner's "forest of symbols" when I touched on culture. This sounds like poetry rather than mathematics, but thanks, famously, to Nelson Goodman and his memetic descendants we now have hard-nosed theories of symbols. When combined with other recent ideas, it may promise a next stage of major progress in gene-culture mathematical studies. Certainly, if I have any complaint at all about this book, it is the purely minor one that RB themselves have little to say – beyond a chaste "there is still much to explain" – about their view of what lies ahead as they review their thirty years of effort in this field.

Nevertheless, researchers and students in the human sciences, sociobiology, evolutionary psychology, and GCC research will welcome this book, which compiles and synthesizes results heretofore available only by digging through the scattered specialist literature. The material is important both for what it accomplishes and for all that it leaves undone. Well showcased are merits of simple mathematical models as an aid to exploring specific evolutionary effects in the gene-culture linkage. Also on display are the potential limitations of the current mathematical approaches and their underlying premises, but in this Boyd and Richerson do not stand alone. All of us GCC modelers share in them. Changes are needed before there can be a next striking wave of research innovation in this subject.

Why is this so? In a nutshell, it is because we GCC modelers have been too content to labor in the shadow of 19th century physics and applied mathematics, and so to fit our needs and conceptions into that axiomatic frame, instead of building one that fits gene-culture coevolution and evolutionary psychology *ab initio*. A first step, by means of old frameworks well proven in terms of prior applications, is reasonable. But after a quarter-century the big questions about the evolution of mind and culture are as elusive as ever, and new mathematical frameworks are needed.

Admittedly, it is in part better math allowing better experimental measurement and testing of the models. But only in part: The deductions of Cavalli-Sforza and Feldman about the respective roles of horizontal, oblique, and vertical meme transmission routes, the inferences of Lumsden and Wilson about the amplification of developmental genetic changes into shifts of large-scale cultural patterns, the conclusions of Boyd and Richerson about the evolution of cooperation and cultural group selection in biocultural populations – these are examples of predictions stimulated by the mathematical work so far. So already there are key inferences about what we should be seeing in the human mind and the biocultural record. For the most part, experimental science has not yet caught up to them. The program of Cosmides, Tooby, and their colleagues is an outstanding example of the progress that can be achieved in evolutionary psychology when formal models and experiment are in synch.

The need for predictions is not the driving force for radical progress in GCC mathematics. The force I see is internal to the subject itself, given what we (want to) know about human nature. Evolution is about change, and in the mathematical language of approaches to date, change is about “dynamics,” i.e. the solution of differential equations or their ilk that make up the “equations of motion” for the evolutionary process. BR are fond of discrete time-step equations for the dynamics in their models, rather than the continuous axis of temporal change in differential equations, but the point is the same: discrete or continuous, deterministic or stochastic, we want the equations of motion; they are our Rosetta Stone for translating the pattern of evolutionary forces into predictions of the evolutionary path the population tracks, in response to those forces.

So far so good, but then we turn to the kinds of mathematical arenas, invented between the time of Newton and the time of Einstein, used to represent this evolutionary change. By and large these arenas are sets of elements, each element representing or “marking” a possible state of the evolving system. Usually the elements are labeled with numbers, or strings of numbers, that demarcate them quantitatively. The equations of motion specify the rates (or something similar) at which any one state gives way to others accessible from it, and so on through each moment of time in the evolutionary progression. The set of elements often has a metric, or natural measure of distance, associated with it, which allows us to say when a state has changed a little or a lot, and by how much. When suitably posed, the metric can be read as equipping the set of elements with geometric properties.

These properties are intrinsic to the evolutionary change and can be freed from the arbitrary manner in which we might map, or link, the state elements to their numerical indices. It is then natural to think of such a set, equipped with a natural geometry, as a “space” of “points,” each point marking a state, and of the evolutionary change as tracing out a path or trajectory though this space. So, for example, a state element or point represents a population in which the frequency of a gene variant has a specific value and that of a meme variant also has a specific value. Points with slightly different frequency values are nearby in the space. The equations of motion connect the points in an axiomatic game of “join-the-dots,” to predict which point will follow which as the population evolves. A glance in any textbook about mathematical population genetics, ecology, or neural network theory, for instance, will reveal endless content based on this general point of view.

Equations tracing evolutionary paths down to quantitative precision sounds pretty good, and indeed they certainly are not bad. Equations of motion on such spaces are made from the get-go for being solved, at least in approximate numerical terms. The solutions are therefore quantitative predictions about what happens as the evolutionary process unfolds. Deterministic and probabilistic changes can be handled, as can discrete as well as continuous alterations. No sweat. The predictions either are confirmed by experiment, or not, so such models are not fly-by-night stories that can elude scientific scrutiny by, chameleon-like, switching their intended meaning at the last minute.

But let’s take a closer look. Tagged to each of those state points is a number or a set of numbers. Numbers are good when we want to count things, and points are good when we have reason to believe the states of our system reduce to geometric singularities. Sometimes we do want to count things – the number of variants of a kind of gene variant (sickle cell or not?) or meme (conservative or liberal today?), for instance, or of a certain style of clay pot. Without doubt this is the kind of “actuarial” dynamics in which GCC mathematics has excelled since the early 1980s monographs by Lumsden and Wilson (1981) and Cavalli-Sforza and Feldman (1981). It fills the BR book from cover to cover.

At other times, however, we might want to do more than count pots. Psychologists in particular, I think, need take a guarded view of mathematical models framed as state changes over spaces of numerically labeled points. Why? As I see it, we cannot have an adequate mathematics of gene-culture coevolution without an adequate mathematics of behavior. How could we, since to feign otherwise would merely be to sweep the effects of learning, choice, and decision into “preference functions” that simply hide behavior inside some innocent looking mathematical parameters? Similarly for culture, whatever that esoteric thing finally turns out to be in mathematical terms. In the psychological part of the problem we need mathematical constructs that express specific conditions of the evolved embodied mind. It may prove ill advised to squeeze such a representation down to a single number or number string.

To see what I am getting at here, let us do a short experimental run, in which you gather data on my current mood and on one or more items in my store of declarative knowledge. In Run 1 you get the mood data first and then the declarative item, while in Run 2 you get the declarative item first and then the mood item. Run 1: if you ask me how I feel just now, I’ll report the positive feelings allied to the pleasant task our Editor has set me in composing this essay on and around an excellent book. If you then ask me what is the size of the tumor now growing in my kidney, I’ll tell you such-and-such a diameter based on my recent medical imaging scans. So you get the positive mood datum and the tumor size datum. Run 2: We reverse the order of the queries. You ask me about the tumor size first and get the same number as in Run 1. But now I am thinking about the tumor and likely to start ruminating on the future, so when you ask me about my mood you will get a more somber report than in Run 1. Bottom line: the order in which you have made observations matters.

From the standpoint of mathematical theory there is a lot going on in a thought experiment like this, which it is well beyond the scope of this assignment to unpack. I think, however, that in part such considerations are telling us that the algebra of observables, defined over the space of states characterizing mind and culture, does not fully commute. Numbers, and the usual functions of numbers, do commute, so they cannot be the whole mathematical story about us. Other mathematical objects, however, are more suited to the non-commutative job. For example, the net-like structure of a brain circuit or a semantic network or a pattern of cultural meaning has a natural associative pattern, i.e. a net-like pattern in which elements are interconnected.

The mathematical objects that quantify such patterns of associative connection are matrices, rather than single numbers. Matrices do not in general commute. Imagine then a biocultural space in which, as we shrink down and zoom in on single states, the view resolves not into the point singularities of current conception, but into mathematical objects perhaps akin to matrices. Mathematicians are exploring such spaces as homes for generalized concepts of our familiar geometric theorems; the next few years will no doubt see their further extensions into human evolutionary science. I for one would find a mathematics of human evolution that short-changed reason, or passion, or both, quite uninteresting.

There is a further troubling feature of these mathematical spaces used to house models of GCC. Once again it descends from the well established needs of the physical and engineering sciences. We saw above that, at each point of the space, there is a number or collection of numbers labeling the state of the system. If we have a hurtling rocket, for example, the numbers might label the missile's current position, its velocity, and its angular heading relative to the fixed stars. Similarly, in an evolving gene-culture system the numbers might label the current frequencies of the genes and memes we are tracking. The point is that, once determined by the specifications of the dynamics problem, this list of traits does not change.

Consider, however, a society in which there is innovation (the creation of new memes, and the spread of said memes to others in the population; all human cultures have this). From the mathematical point of view, after the innovation event there is something new to be counted and tracked, and it is not in the list of numbers attached to any of the points in the model's evolutionary space. A new equation has suddenly appeared in our list of equations of motion. The mathematical spaces used in current work don't like this. They are built for problems in which the equations of motion do not morph and mutate and jump around. That's okay for the trajectories prescribed *by* the equations of motion – they can twist and turn – but not for the population of motion equations themselves. This "tight bind of the fixed dimensions" will have to be circumvented if mathematical treatments of gene-culture coevolution are to become nontrivial, i.e. if they are to predict what is not already obviously built into the model.

Mathematicians and physical theorists have cooked up some interesting possibilities for spaces that don't mind having their dimensions and equations of motion come and go. They have imposing names, like "Fock space" and "superspace," but tend to make use of a trick which makes them of little use to the psychologist who takes creativity, innovation, and other dynamics-busting traits of human nature seriously: they hide the problem within an infinitude of possibilities worked out in advance by the modeler. This really is of no use at all to us, since one wants a biocultural mathematics with room for the unforeseen and its consequences, i.e., of creativity and innovation.

Indeed, I do not think we can score significant further advances in biocultural mathematics unless we create a deep mathematics of mind. What ties genes to culture in human history except the activities of mind? That is, surely, equivalent to saying the GCC problem contains the mind problem, which for our human species contains the problem of consciousness (Cs). What to do about Cs in sociobiology and evolutionary science is beyond the scope of this essay, but I will note I have considered the Cs issue at somewhat more length elsewhere (Lumsden, 2005) and conclude that all is not lost (maybe) – especially if those following a mathematical approach are willing to consider still more general spaces in which the ground symbols stretch past numbers and begin expressing the nature of cultural and mental things.

Also in play, of course, is Roger Penrose's unpopular idea about Cs noted above (Penrose, 1994). If I understand Professor Penrose, this is the possibility that Cs may be impervious to any current means of scientific calculation, prediction, or understanding because Cs – at least in its manifestations of creativity and subjective self awareness – entails properties of our Universe currently beyond the ken even of our most up to date quantum theories of nature. Its explanation, Penrose anticipates, will require a revolution in physics, with all the shock waves that may send through the natural and human sciences.

Such caveats still fall within the purview of the third premise I promised you earlier. This premise states that gene-culture coevolution is *compressible,* i.e. that it *is* amenable to explanations more concise than a straight chronicle or full narrative record of all of biocultural history itself. Science, from the point of view championed by this premise, is the art of the highly compressible, in other words of apprehending those parts of existence that can be wrapped up in short explanations. Otherwise, simple explanations of apparently complex things are a contradiction in terms.

Physics currently reigns supreme in this regard, having found a mode of mathematical explanation in which the paths of footballs, planets, stars, and galaxies all follow from a few lines of mathematical equations. Indeed, we hear physical scientists talk about the imminent arrival of a "theory of everything," but given my remarks on the downside of vulgar reductionism, we must be prepared for a restrained reading of the term "everything," even if current efforts to blend quantum theory and Einstein's gravitation work out. We have seen above how influential the mathematical tools championed by physical theorists have been in other fields, including the modeling of gene-culture coevolution. We have also seen a few of the reasons why GCC may require more than such tools can at present deliver. The human sciences are, I think, absolutely justified in demanding to know why a premise of high compressibility should apply to their subject matter. Just how concise *can* we get in these disciplines? Are the evolved mind and its gene-culture history, or at least their gist, tied up in a few lines of equations? Or are they their own shortest possible explanation?

The only straight answer is that, at present, no one yet knows the epistemic compressibility of minds, human evolutionary history, and gene-culture coevolution.

A long time ago I estimated that the brain and mind are highly incompressible from the *gene's* point of view (Lumsden and Wilson, 1981), but that is a different story. We are reflecting on the goals of scientific explanation, not on the role hereditary molecules play in development.

The mathematician Gregory Chaitin, (1987) has developed an ingenious way of thinking about this problem, which is very helpful here. The idea appears baldly simple but can be shown to have remarkable consequences. In step one of Chaitin's approach, we assess a measure of the size of our original description or depiction of the phenomenon. Let that size be *D*. In step two, we determine the size of the smallest computer program capable of producing the description *D* as output. Let that size be *P*. (In what follows I will also use the symbols *D* and *P* to refer to the description and the program themselves, as well as to their sizes.)

In the final step we just figure the ratio *P*/*D*. If it is considerably less than one, we have distilled *D* into a very concise computer program (our "model" or "theory" of *D*) and say that *D* is highly compressible. We have squeezed it down into a tiny mathematical formulation. If *P*/*D* is no different from unity, then our description *D* is quite incompressible. *D* is then its own shortest possible explanation. For example, if *D* is the enormously long string of numbers giving the orbital position of our Moon since first it formed and settled into its path round the Earth, an astronomer could regenerate *D* with a computer code *P* containing the concise equations of celestial mechanics. A *D* like that is highly compressible. For things like the mind and its evolution, however, we must ask if there is any reason to think *P*/*D* might be small, and research using simple mathematical models thus motivated.

This sounds like a very abstract problem. As a problem asking about mind and culture in general, it certainly is abstract, and no doubt difficult, at least for now. But a specific example, which can readily be generalized, can help us see what is involved in posing such questions in specific terms. For example, I am very attached to Homer's *Iliad* but, my archaic Greek being what it is, I have a shelf full of English translations, all well-thumbed. Some are more concise than others. The full text of Alexander Pope's rhyming couplets and notes (1996/1715–1720), for instance, considerably outweighs Stanley Lombardo's (1997) sinewy English vernacular. Where on the shelf of variously sized translations is the "real" *Iliad*? To streamline the formalities let us take as the *Iliad* one of the standard editions in Greek (Monro and Allen, 1920, for example) our master translators use in rendering Homer's artistry into English. Such an *Iliad* is a long poem – almost 16,000 lines by many counts. A glance through my English translations gives, say, roughly ten words per each of these 16,000 lines, for 160,000 words of Homeric action. That's an approximate word count but will do here. Now let those words, in sequence from first to last, be our *D*, the initial description or cultural object. Since the *Iliad* is, canonically, a founding text of Western culture, it makes a useful touchstone indeed.

Is this *D* compressible, and so recoverable from a small mathematical model? If it is, then we mean something like the following is true: There exists a mathematical equation Φ_{Iliad} with two properties:

- If k is a word from the *Iliad*, i.e. if k is the k-th word from D, then $\Phi_{\text{Iliad}}(k) = k + 1$, i.e., the next word in the text, for k running from the first word right through to the next-but-last word of the epic. So the first few steps in using this equation would be (my crude rendering into English):
  - Φ_{Iliad} (*Sing*) = *oh*
  - Φ_{Iliad} (*oh*) = *ye*
  - Φ_{Iliad} (*ye*) = *Goddess,*
  - Φ_{Iliad} (*Goddess,*) = *Rage*
  - and so on.
- $\Phi_{\text{Iliad}}(k)$ requires far less than 160,000 words to write down.

Taken separately, these two criteria are each easy to fulfill. For example, the formula $\Phi(k) = k^2 + 1$ is very concise, but will not generate the *Iliad*. On the other hand, we could readily devise an exactly accurate $\Phi_{\text{Iliad}}(k)$ by loading Homer's text into a computer program, then just having the program print out word $k + 1$ whenever we input word k. This would give us a Φ_{Iliad} all right, but it would be even longer than the original work, *not* shorter – this "easy" Φ_{Iliad} would contain not only all of the *Iliad* text D we started with, but the lines of code for printing it out too. So its size P would be even greater than the *Iliad*'s and we would get no compressibility at all. (A strict approach might encourage us also to include the size of the computer's operating system, compiler, printer drivers, and so on in estimating the size of our Φ_{Iliad}, in which case the amount of spare room we are left with to write down a working Φ_{Iliad} is going to be quite tight indeed.) Meeting both criteria at once – completeness and concision – is going to be hard.

We can appreciate the magnitude of the task by considering the *Iliad* against the background space of all possible texts of the same length. My morning paper has reported that the English language is expected to officially assimilate its millionth word by summer or autumn 2006 (Kesterton, 2006; Peritz, 2006). One count currently stands at 986,120 words. Let us take the store of English as an even million, or 10^6, and go back to D, our 160,000 words of a hypothetical translated *Iliad* text. Consider again the operation of the mathematical equation Φ_{Iliad}. At each step k for English readers, it must pick out the one "right" word from a million possibilities, and do so 160,000 times in sequence. The possibilities grow very big very fast. If there are 10^6 options at each step k, then there are $(10^6)^{160{,}000} \sim 10^{1{,}000{,}000}$ alternative texts 160,000 words long in a language of a million words, ignoring the niceties of punctuation and whatnot. Φ_{Iliad} must zero in on the one matching D. Even at the first word (*Sing*, in my take above), there are a million possible choices for where to go next, then a million beyond each of those million choices, and so on.

The psycholinguists could trim the size of this space by imposing constraints of grammatical structure, but many alternatives would remain. How big is $10^{1,000,000}$? It's big compared to what physical (as opposed to cultural) nature presents us. The Universe is roughly 15 billion years, or 5×10^{16} seconds, old. The Planck time – considered by some cosmologists as the increment below which time's passage becomes quantum-discrete rather than a continuous flow – is some 10^{-43} seconds. So roughly 10^{60} steps of Planck time (all the time there has been thus far) have elapsed since the Big Bang. That number is essentially zero when compared to numerical behemoths like $10^{1,000,000}$.

These kinds of number games can of course be taken too far. The intent of the above is to give us some appreciation of the magnitude of the cultural diversity lingering behind objects like the *Iliad*, apparent to us once we start putting the issue in mathematical terms. This in turn allows us to appreciate the targeting precision needed from mathematical models Φ_{Iliad} as they regenerate D from their equation P. It is quite clear, however, that targeting is not the issue in itself. Take the $\Phi(k) = k^2 + 1$ equation we dealt with above. Give it a real number k to start, and this little formula will happily pick exactly the right subsequent value (i.e., $k^2 + 1$) from the continuum of the real number line. That continuum holds a range of options (of magnitude c, the cardinality of the real numbers) that dwarfs magnitudes like $10^{1,000,000}$, but our little $\Phi(k)$ selects the right trajectory each and every time we give it the starting k. The real issue whether a fully accurate equation $\Phi(k) = \Phi_{\text{Iliad}}(k)$ can have $P << D$ to boot.

Let us think about this in psychological terms. Since Φ_{Iliad} essentially "authors" the *Iliad* once we feed it the initial prompt "*Sing*," we are in effect asking whether an equation describing what went through Homer's mind as he created his 16,000-line epic can itself be written down in far less than 16,000 lines of computer code, i.e. in $P << D$. The real "Homer" may have been one creative genius or a group of bardic masters spun out over time and place, but the point about compressibility stands regardless. In fact the *Iliad* as our example D is all the more interesting in this regard, since its origins in an oral culture of bardic performance suggest the existence of a $\Phi_{\text{Iliad}}(k)_{\text{Oral}}$ – a learnably concise set of rules for composing an *Iliad* in one's head on the fly, as it were – a folk theory of the *Iliad* – capable of oral transmission from bard to bard, in contrast to the rote memorization of a 16,000-line poem (for which there is very little evidence).

The *Iliad's* many levels of modular organization are commonly thought to be signs of a more concise plan or narrative blueprint anchoring the fully elaborated recitations, each a unique but valid "*Iliad*." The equation $\Phi_{\text{Iliad}}(k)_{\text{Oral}}$ would stand in contrast to a mathematical equation $\Phi_{\text{Iliad}}(k)_{\text{Text}}$ capable of outputting exactly and fully the complete "official" *Iliad* text (e.g. Monro and Allen, 1920), which was achieved in a mammoth editorial effort by scholars in the centuries after Homer – and, of course, the fruit of a literate, rather than strictly oral, system of culture transmission. In view of the completeness and precision demanded of $\Phi_{\text{Iliad}}(k)_{\text{Text}}$ (the full exact text) compared to $\Phi_{\text{Iliad}}(k)_{\text{Oral}}$ (a recitation valid to listeners of the time) we would not be surprised if the former equation's length greatly exceeded the latter's, perhaps even compared to the size *D* of the full text itself.

Literate culture, with its received texts, is then radically less compressible, for both learner and for mathematical model, than is oral culture. If Penrose's conjecture is right, the foregoing considerations are not even well posed, since Φ_{Iliad} points to something not computable by any notions of science or mathematics we now have. Theories of gene-culture coevolution will then have to await (or help provoke) seismic displacements in mathematical physics. And yet one does not have to believe in the need for such revolutions to doubt the premise that, for mind and culture, high compressibility and the concision of general laws will prevail. Creativity simply may be its own shortest possible description.

For example Deep Blue, the famous IBM computer that took on world champion chess grand master Garry Kasparov in 1996 and, in upgraded form, again in 1997 (http://www.research.ibm.com/deepblue/) is reported to have used some 8,000 terms in the evaluation function (its Φ_{Chess}) by which possible next moves on the chess board were evaluated (http://en.wikipedia.org/wiki/Deep_Blue). Deep Blue also had an enormous chess move database and hardware designed especially for generating vast numbers of chess piece positions (some 200,000,000) per second. But let us for simplicity ignore the database and specialized hardware, and just use $P \sim 8{,}000$. And yet *D*, the complete narrative of the battle fought out on the chess board in these games, requires only about 50 short lines per game (see http://www.research.ibm.com/deepblue/watch/html/c.shtml), giving $P/D \sim 400 >> 1$, i.e. hugely incompressible. Is a game of chess, even a grand master game of chess, more creative than the *Iliad*, or will the challenges to mathematical science posed by such manifestations of mind and culture be even greater? Until firmer results are available, mathematical models will continue the practical strategy of their experimental counterparts, namely assuming that the simplest conceivable formulations are helpful and then testing them with hard data.

It is said that Achilles, Iliadic champion of the Greek invaders, sat in a funk on the Trojan shore, gazing out on the wine dark sea as carnage raged and bronze clad warriors perished in the dunes behind him, the sand red with their blood. Epic history does not report much about what Achilles saw as he looked out, apart from his goddess mother rising from the waves in answer to his prayerful behest. For us, regardless of how the Penrose Conjecture, compressibility lemmas, and new methods play out, it is clear that a small band of very real scholars has established a beachhead on the shores of psychohistory. From this vantage point we can already see that much of the apparatus used in the first assault lies disused or in wreckage, ready to be superceded by new ideas. That seems to me as it should be. Science is about the future, as well as the now and the past (Lumsden, 2004). Boyd and Richerson have been pioneers in grounding biocultural studies as a young but thriving science in which mathematical theory is a partner to empirical discovery. Their book is of permanent value in gauging the view back and in considering what will come in the years ahead.

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